$$p^{\mu'}_{TOT} = \left( \frac{E'_{tot}}{c}, \overrightarrow{p'}_{tot} \right) \tag{3.1.4}$$

$E'_{tot}$ = total energy of after collision

$\overrightarrow{p'}_{tot}$ = total momentum of after collision

$$E'_{tot} = m_{\Lambda}c^2 + m_{\pi^-}c^2$$

$$E'_{tot} = (m_{\Lambda} + m_{\pi^-})c^2 \tag{3.1.5}$$

By substituting equation (3.1.5) into equation (3.1.4), it is following equation

$$P^{\mu'}_{TOT} = \left( \frac{(m_{\Lambda} + m_{\pi^-})c^2}{c}, \vec{P}'_{tot} \right) \tag{3.1.6}$$

Since all particles are at test centre of mass frame.

$$\vec{P}'_{tot} = \vec{P}'_{\Lambda} + \vec{P}'_{\pi^-}$$

$$\therefore \overrightarrow{p'}_{tot} = 0$$

From equation (3.1.6), becomes

$$p^{\mu'}_{TOT} = \left( \frac{(m_{\Lambda} + m_{\pi^-})c^2}{c}, 0 \right) \tag{3.1.7}$$

Now, $p^{\mu}_{TOT} \neq p^{\mu'}_{TOT}$ , since invariant $p_{\mu TOT}\, p^{\mu}_{TOT}$ and $p'_{\mu TOT} p^{\mu'}_{TOT}$ are equal.

$$p_{\mu TOT} p^{\mu}_{TOT} = p'_{\mu TOT} p^{\mu'}_{TOT}$$

$$\left(p^{\mu}_{TOT}\right)^2 = \left(p^{\mu'}_{TOT}\right)^2 \tag{3.1.8}$$

By taking squaring both sides equation (3.1.3) and equation (3.1.6),

$$\left(p^{\mu}_{TOT}\right)^2 = \left( \frac{E + m_n c^2}{c}, \vec{p}_{K^-} \right)^2$$

$$\left(p^{\mu'}_{TOT}\right)^2 = \left( \frac{(m_{\Lambda} + m_{\pi^-})c^2}{c}, 0 \right)^2 . \tag{3.1.9}$$

According to equation (3.1.8),

$$\left(p^{\mu}_{TOT}\right)^2 = \left(p^{\mu'}_{TOT}\right)^2$$

$$\left( \frac{E + m_n c^2}{c} \right)^2 - \overrightarrow{p'_{K^-}} = \left( \frac{(m_{\Lambda} + m_{\pi^-})c^2}{c} \right) - 0 \tag{3.1.10}$$

Multiplying $c^2$ both sides equation of equation (3.1.10),

$$E^2 + 2Em_n c^2 + m_n^2 c^4 - p_{K^-}^2 c^2 = (m_\Lambda + m_{\pi^-})^2 c^4 \qquad (3.1.11)$$

The standard relation is

$$E^2 = p_{K^-}^2 c^2 + m_{K^-}^2 c^4 \qquad (3.1.12)$$

By substituting equation (3.1.12) into equation (3.1.11), it is following equation as

$$p_{K^-}^2 c^2 + m_{K^-}^2 c^4 + 2Em_n c^2 + m_n^2 c^4 - p_{K^-}^2 c^2 = (m_\Lambda + m_{\pi^-})^2 c^4$$

$$2Em_n c^2 = (m_\Lambda + m_{\pi^-})^2 c^4 - m_{K^-}^2 c^4 - m_n^2 c^4$$

$$E = \frac{(m_\Lambda + m_{\pi^-})^2 c^4 - m_{K^-}^2 c^4 - m_n^2 c^4}{2m_n c^2}$$

$$E = \frac{(m_\Lambda + m_{\pi^-})^2 - m_{K^-}^2 - m_n^2}{2m_n} c^2$$

$$E = \frac{M^2 - m_{K^-}^2 - m_n^2}{2m_n} c^2$$

Where, $M = m_\Lambda + m_{\pi^-}$, E= threshold energy.

This is general formulation of threshold energy of $K^- + n \to \Lambda + \pi^-$ reaction. It is obtained 238.9 MeV for above reaction.

We also calculated the threshold energy of $\pi^+ + n \to \Lambda + K^+$ reaction and $K^- + P \to \Xi^- + K^+$ reaction. It is obtained 898.07 MeV and 1156.62 MeV.

## 3.2 Newton-Raphson Method

Consider a graph of $f(x)$ as shown in Figure. Let us assume that $x_1$ is an approximate root of $f(x)=0$. Draw a tangent at the curve $f(x)$ at $x = x_1$ as shown in the figure. The point of intersection of this tangent with the x-axis gives the second approximation to the root. Let the point of intersection be $x_2$. The slope of the tangent is given by

$$\tan \propto = \frac{f(x_1)}{x_1 - x_2} = f'(x_1) \qquad (3.2.1)$$

Where $f(x_1)$ is the slope of $f(x)$ at $x = x_1$. Solving for $x_2$ we obtain

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} \qquad (3.2.2)$$

This is called the Newton-Raphson formula.

The next approximation would be

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} \qquad (3.2.3)$$

In general,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \quad (3.2.4)$$

This method of successive approximation is called the Newton-Raphson method. The process will be terminated when the difference between two successive values is within a prescribed limit.

The most widely used of all methods for finding roots is the Newton-Raphson method.

### 3.3 Calculation of Momentum Transfer for Various Reactions

By law of conservation of energy,

$$E_K + E_n = E_\pi + E_\Lambda \quad (3.3.1)$$

$$\sqrt{P_K^2 + m_K^2} + \sqrt{P_n^2 + m_n^2} = \sqrt{P_\pi^2 + m_\pi^2} + \sqrt{P_\Lambda^2 + m_\Lambda^2} \quad (3.3.2)$$

$$\sqrt{P_K^2 + m_K^2} + m_n = \sqrt{p_\pi^2 + m_\pi^2} + \sqrt{P_\Lambda^2 + m_\Lambda^2} \quad (3.3.3)$$

By law of conservation of momentum,

$$\vec{P}_K = \vec{P}_\pi + \vec{P}_\Lambda \quad (3.3.4)$$

$$\vec{P}_\pi = \vec{P}_K - \vec{P}_\Lambda \quad (3.3.5)$$

$$\vec{P}^2{}_\pi = \vec{P}_K^2 + \vec{P}_\Lambda^2 - 2\vec{P}_K \vec{P}_\Lambda \cos\theta \quad (3.3.6)$$

$$\sqrt{P_\pi^2 + m_\pi^2} + m_n = \sqrt{P_K^2 + m_K^2} + \sqrt{P_\pi^2 + P_K^2 - 2P_\pi P_K \cos\theta + m_\Lambda^2} \quad (3.3.7)$$

$$f(P_\Lambda) = \sqrt{P_K^2 + m_K^2} + \sqrt{P_\pi^2 + P_K^2 - 2P_\pi P_K \cos\theta + m_n^2} - \sqrt{P_\pi^2 + m_\pi^2} - m_n \quad (3.3.8)$$

$$\frac{df(P_\Lambda)}{dP_\Lambda} = \frac{1}{2}\left(P_\Lambda^2 + m_\Lambda^2\right)^{-1/2} \cdot 2P_\Lambda + \frac{1}{2}\left(P_\Lambda^2 + P_K^2 - 2P_\Lambda P_K \cos\theta + m_\pi^2\right)^{-1/2}\left(2P_\Lambda - 2P_K \cos\theta\right) \quad (3.3.9)$$

$$\frac{df(P_\Lambda)}{dP_\Lambda} = \frac{P_\Lambda}{\sqrt{P_\Lambda^2 + m_\Lambda^2}} + \frac{(P_\Lambda - P_K \cos\theta)}{\sqrt{P_\Lambda^2 + P_K^2 - 2P_\Lambda P_K \cos\theta + m_\pi^2}} \quad (3.3.10)$$

$$E_\pi = \sqrt{P_\pi^2 + m_\pi^2} \quad (3.3.11)$$

$$\sqrt{P_\pi^2 + m_\pi^2} = \sqrt{P_\Lambda^2 + P_K^2 - 2P_\Lambda P_K \cos\theta + m_\pi^2} = E_\pi \quad (3.3.12)$$

$$\frac{df(P_\Lambda)}{dP_\Lambda} = \frac{P_\Lambda}{E_\Lambda} + \frac{P_\Lambda - P_K \cos\theta}{E_\pi} \quad (3.3.13)$$

According to Newton-Raphson method, we can write the following equation.

$$p_{\Lambda 1} = p_{\Lambda} - \frac{f(p_{\Lambda})}{f'(p_{\Lambda})} \quad (3.3.14)$$

momentum transfer for $\pi^{+} + n \rightarrow K^{+} + \Lambda$ reaction,

$$\frac{df(P_{\Lambda})}{dP_{\Lambda}} = \frac{P_{\Lambda}}{E_{\Lambda}} + \frac{P_{\Lambda} - P_{\pi}\cos\theta}{E_{K}} \quad (3.3.15)$$

$$p_{\Lambda 1} = p_{\Lambda} - \frac{f(p_{\Lambda})}{f'(p_{\Lambda})} \quad (3.3.16)$$

momentum transfer for $K^{-} + p \rightarrow K^{+} + \Xi^{-}$ reaction,

$$\frac{df(P_{\Xi})}{dP_{\Xi}} = \frac{P_{\Xi}}{E_{\Xi}} + \frac{P_{\Xi} - P_{K^-}\cos\theta}{E_{K^+}} \quad (3.3.17)$$

According to Newton-Raphson method, we can write the following equation.

$$p_{\Xi 1} = p_{\Xi} - \frac{f(p_{\Xi})}{f'(p_{\Xi})} \quad (3.3.18)$$

We calculate momenta of kaon and lambda. We solved numerically equation (3.3.14), (3.3.16) and (3.3.18) to obtain lambda and xi momentum for various incident momentum of different reactions by using FORTRAN CODE.

## IV. Results and Discussions

We calculated threshold energies of $K^{-} + n \rightarrow \Lambda + \pi^{-}$, $\pi^{+} + n \rightarrow \Lambda + K^{+}$ and $K^{-} + p \rightarrow \Xi^{-} + K^{+}$ reactions. The threshold energies are obtained 238.9MeV and 898.07MeV and 1156.62MeV respectively. We calculated momentum transfer for various incident momentum in $K^{-} + n \rightarrow \Lambda^{-} + \pi^{-}$ reaction by using Newton-Raphson method. These results are shown in table (4.1). It is observed that the momentum of lambda for various incident momentum is always below the Fermi level (270MeV). Therefore lambda can exist in the nuclei in this reaction. We also calculated momentum transfer for various incident momentum in $\pi^{+} + n \rightarrow K^{+} + \Lambda$ reaction. The results are shown in figure (4.1) and table (4.2). In this reaction, the momentum transfer for greater than incident momentum 1800MeV. The momentum $K^{-} + p \rightarrow K^{+} + \Xi$ reaction. This reaction is above the Fermi-level and hyperon $\Xi$ cannot exist in the nucleus.

| Incident momentum(MeV/c) | $p_\Lambda$(MeV/c) $\pi^+ + n \rightarrow K^+ + \Lambda$ | $p_\Xi$(MeV/c) $K^- + P \rightarrow K^+ + \Xi^-$ |
|---|---|---|
| 800 | 223.86 | 634.99 |
| 900 | 540.32 | 876.27 |
| 1000 | 430.94 | 760.13 |
| 1100 | 386.25 | 653.93 |
| 1200 | 358.01 | 579.27 |
| 1300 | 337.82 | 550.62 |
| 1400 | 322.43 | 532.44 |
| 1500 | 310.21 | 519.75 |
| 1600 | 300.22 | 510.4 |
| 1700 | 291.88 | 503.23 |
| 1800 | 284.78 | 497.59 |
| 1900 | 278.67 | 493.06 |
| 2000 | 273.34 | 489.34 |

Table (4.1)momenta of incidient particle and product lambda for $K^- + n \rightarrow \pi^- + \Lambda$ and $K^- + p \rightarrow \Xi^- + K^+$ reaction

| Incident momentum (MeV/c) | $p_\Lambda$(MeV/c) $K^- + n \rightarrow \pi^- + \Lambda$ |
|---|---|
| 0 | 253.25 |
| 510 | -5.13 |
| 1020 | 75.70 |
| 1530 | 110.71 |
| 2040 | 129.72 |

Table (4.2) momenta of incidient kaon and product lambda for $K^- + n \rightarrow \pi^- + \Lambda$ reaction